

Supersymmetric Brane-Worlds

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Abstract

We present warped metrics which solve Einstein equations with arbitrary cosmological constants in both in upper and lower dimensions. When the lower-dimensional metric is the maximally symmetric one compatible with the chosen value of the cosmological constant, the upper-dimensional metric is also the maximally symmetric one and there is maximal unbroken supersymmetry as well.

We then introduce brane sources and find solutions with analogous properties, except for supersymmetry, which is generically broken in the orbifolding procedure (one half is preserved in two special cases), and analyze metric perturbations in these backgrounds

In analogy with the D8-brane we propose an effective $(\hat{d} - 2)$ -brane action which acts as a source for the RS solution. The action consists of a Nambu-Goto piece and a Wess-Zumino term containing a $(\hat{d} - 1)$ -form field. It has the standard form of the action for a BPS extended object, in correspondence with the supersymmetry preserved by the solution.

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Introduction

Randall and Sundrum's recent proposal for an alternative to standard Kaluza-Klein (KK) compactification in Refs. [2, 1] has attracted a lot of attention from many quarters: from a phenomenological point of view, it is a new and fresh proposal to understand the hierarchy between gauge and gravitational interactions, while from a purely gravitational point of view it rises many interesting problems concerning the relation between bulk and brane gravitational phenomena. In any case, these models provide a new arena in which one can study new and old problems of Theoretical Physics.

It is worth trying to extend this framework. Here we will present generalizations of the Randall-Sundrum (RS) scenario which could be used as alternatives to KK compactification. They are solutions of the Einstein equations with arbitrary cosmological constant in \hat{d} dimensions and lead to $d = (\hat{d} - 1)$ -dimensional metrics which solve the Einstein equations with arbitrary d -dimensional cosmological constant. They have a property which one should require of any framework with extra dimensions: when the d -dimensional metric is maximally symmetric (and, therefore, is the lower-dimensional vacuum) the corresponding \hat{d} -dimensional metric is also maximally symmetric (and, therefore, the upper-dimensional vacuum). This holds in any consistent standard KK compactification: vanishing matter fields and Minkowski metric in lower dimensions (the d -dimensional vacuum) correspond to Minkowski times a torus metric in upper dimensions (the \hat{d} -dimensional vacuum). The same can be said of supersymmetry although there are subtleties that in many cases will make impossible to define lower-dimensional supersymmetry.

In order to exploit these “bulk” solutions for dimensional reduction, we introduce brane sources, find the modified solutions and study the dynamics of gravitons in the new backgrounds. We also find the effective gravity actions and Newton constants in lower dimensions and study supersymmetry on the brane-worlds.

In finding the solutions with branes, we have to allow for cosmological constants that are piecewise constant functions, a fact which comes naturally when dualizing it. Therefore we consider gravity coupled to a volume-form field strength and coupled to a generic $(\hat{d}-2)$ -brane action.

1 Bulk Solutions

We are interested in “warped metrics” of the form⁴

$$d\hat{s}^2 = a^2(y) ds^2 - dy^2, \quad ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu, \quad (1.1)$$

solving the equations

$$\hat{R}^{\hat{\mu}\hat{\nu}} = \hat{\Lambda} \hat{g}^{\hat{\mu}\hat{\nu}}, \quad R^{\mu\nu} = \Lambda g^{\mu\nu}, \quad (1.2)$$

⁴We work in arbitrary dimension \hat{d} with mostly minus signature. All \hat{d} -dimensional objects carry hats. We choose $x^{\hat{d}-1} \equiv y$ as the spacelike holographic coordinate and thus, we split the $\{\hat{x}^{\hat{\mu}}\} = \{x^\mu, y\}$. Unhatted objects are $d (= \hat{d} - 1)$ -dimensional.

where $\hat{\Lambda}$ and Λ are respectively the \hat{d} and d -dimensional cosmological constants whose signs are, in principle, arbitrary. We define \hat{g} and g by

$$\hat{g}^2 = -\frac{\hat{\Lambda}}{(\hat{d}-1)}, \quad g^2 = -\frac{\Lambda}{(d-1)}. \quad (1.3)$$

The solutions fall into two classes:

$$1. \quad \hat{\mathbf{g}} \neq \mathbf{0} \quad a(y) = \frac{1}{2} \sqrt{\pm g^2 / \hat{g}^2} (e^{\hat{g}y} \pm e^{-\hat{g}y}), \quad (1.4)$$

where the sign has to be chosen such as to make $a(y)$ real. This is always possible except for the case $\hat{g} \in \mathbb{I}, g \in \mathbb{R}$. In the other cases we have⁵, with $g \neq 0$

$$(a) \quad \hat{\mathbf{g}}, \mathbf{g} \in \mathbb{R} \quad a = g/\hat{g} \cosh \hat{g}y, \quad (1.5)$$

$$(b) \quad \hat{\mathbf{g}} \in \mathbb{R}, \mathbf{g} \in \mathbb{I} \quad a = ig/\hat{g} \sinh \hat{g}y, \quad (1.6)$$

$$(c) \quad \hat{\mathbf{g}}, \mathbf{g} \in \mathbb{I} \quad a = g/\hat{g} \cos i\hat{g}y. \quad (1.7)$$

In this case, the coordinate y naturally lives in a circle of length $\frac{2\pi}{i\hat{g}}$. With $g = 0$ the only possibility is $\hat{\mathbf{g}} \in \mathbb{R}$ and

$$a = e^{\pm \hat{g}y}. \quad (1.8)$$

$$2. \quad \hat{\mathbf{g}} = \mathbf{0} \quad a = igy, \quad (1.9)$$

which means that we must have $g \in \mathbb{I}$.

The main property of these solutions is that, if $g_{\mu\nu}$ is the maximally symmetric metric in d dimensions with curvature given by Λ , then $\hat{g}_{\hat{\mu}\hat{\nu}}$ is the maximally symmetric metric with curvature given by $\hat{\Lambda}$. The RS solution [2, 1] fits in the $\hat{g} \in \mathbb{R}, g = 0$ case: if $g_{\mu\nu} = \eta_{\mu\nu}$, we have upstairs (locally) anti-De Sitter (aDS). Other possibilities that we are introducing here are: to have either aDS or DS both upstairs and downstairs, to have aDS upstairs and DS downstairs and to have Minkowski upstairs and DS downstairs. The most interesting options (at least from the supersymmetry point of view) are the RS solution and the one with Minkowski upstairs and DS downstairs.

⁵The case in which a is not real can be fixed by Wick-rotating y into a timelike coordinate.

In any dimension, in absence of other fields, the gravitino supersymmetry transformation law will take the form⁶

$$\delta_{\hat{\epsilon}} \hat{\psi}_{\hat{\mu}} = \hat{\mathcal{D}}_{\hat{\mu}} \hat{\epsilon}, \quad (1.10)$$

where $\hat{\mathcal{D}}_{\hat{\mu}}$ is the aDS ($\hat{g} \in \mathbb{R}$) or Lorentz ($\hat{g} = 0$) covariant derivative⁷

$$\hat{\mathcal{D}}_{\hat{\mu}} = \partial_{\hat{\mu}} - \frac{1}{4} \hat{\omega}_{\hat{\mu}}^{\hat{a}\hat{b}} \hat{\gamma}_{\hat{a}\hat{b}} - \frac{i}{2} \hat{g} \hat{\gamma}_{\hat{\mu}}. \quad (1.11)$$

Then, the Killing-spinor equation $\delta_{\hat{\epsilon}} \hat{\psi}_{\hat{\mu}} = 0$ has the following solutions:

$$1. \quad \hat{\mathbf{g}} \neq \mathbf{0} \quad \hat{\epsilon} = \frac{1}{2} (e^{\hat{g}y/2} + \varphi e^{-\hat{g}y/2}) \epsilon_+ + \frac{1}{2} (e^{\hat{g}y/2} - \varphi e^{-\hat{g}y/2}) \epsilon_-, \quad (1.12)$$

where $\varphi = (g/\hat{g})/|g/\hat{g}|$ and where ϵ_{\pm} are two spinors that satisfy

$$(\mathcal{D}_{\mu} \mp \frac{i}{2} g \gamma_{\mu}) \epsilon_{\pm} = 0, \quad (1.13)$$

\mathcal{D}_{μ} being the standard Lorentz covariant derivative and $\gamma_a \equiv \hat{\gamma}_a$. These equations have maximal number of solutions when the d -dimensional space is maximally symmetric.

2. $\hat{\mathbf{g}} = \mathbf{0}$. The solution in this case is any y -independent spinor $\hat{\epsilon}$ satisfying

$$(\mathcal{D}_{\mu} - \frac{i}{2} g \gamma_{\mu}) \hat{\epsilon} = 0, \quad (1.14)$$

where now $\gamma_a \equiv \hat{\gamma}_a \hat{\gamma}_y$. In this case we had to take $g \in \mathbb{I}$ and thus this is the d -dimensional DS covariant derivative. This equation has a maximal number of solutions when the d -dimensional spacetime is DS.

Observe that, although DS supergravity is inconsistent, any pure gravity solution of that theory can be considered a warped compactification of standard (Poincaré) supergravity in one dimension more.

Although we have managed to reduce the \hat{d} -dimensional Killing-spinor equation to a d -dimensional-looking Killing-spinor equation, this does not mean that we have supersymmetry in the d -dimensional space. In the $\hat{g} \neq 0$ case, we cannot have two different signs for g . Keeping only one means keeping either ϵ_+ or ϵ_- , but this truncation is only consistent with d -dimensional Lorentz invariance when $g = 0$ (the RS case). On the other hand in the $\hat{g} = 0$ it seems that there is no problem to have DS supersymmetry. The supersymmetry of the RS solution has also been studied in Refs. [3] and [4]. We will make further comments on their results in the next section.

⁶Depending on the dimension, we will have one or another kind of minimal spinors associated to representations of the gamma matrices with special properties. This will never be an issue in what follows and our results can be adapted to all the cases of interest.

⁷Formally we can also consider the DS case ($\hat{g} \in \mathbb{I}$). DS supergravities do exist even though they are inconsistent as quantum theories.

2 Brane-World Solutions

Now, mimicking Randall and Sundrum we consider the gravity plus brane-sources equations

$$\hat{R}^{\hat{\mu}\hat{\nu}} = \hat{\Lambda}\hat{g}^{\hat{\mu}\hat{\nu}} - \hat{\chi} \left[g^{\rho\sigma} \delta_{\rho}^{\hat{\mu}} \delta_{\sigma}^{\hat{\nu}} - \frac{1}{\hat{d}-2} \hat{g}^{\hat{\mu}\hat{\nu}} (\hat{g}^{\rho\sigma} \hat{g}_{\rho\sigma}) \right] \sum_n T_n \delta(y - y_n), \quad (2.1)$$

$$R^{\mu\nu} = \Lambda g^{\mu\nu}.$$

Although we write cosmological *constants*, we will have to allow for *piecewise constant functions* of y . Then, by making identifications if necessary we can restrict ourselves to a domain in which they are really constant.

With the same Ansatz for the metric Eq. (1.1) these equations reduce to

$$\begin{cases} 0 = a'' + \frac{\hat{\Lambda}}{\hat{d}-1} a + \frac{2\hat{\chi}}{\hat{d}-2} a \sum_n T_n \delta(y - y_n), \\ 0 = (a')^2 + \frac{\hat{\Lambda}}{\hat{d}-1} a^2 - \frac{\Lambda}{d-1}. \end{cases} \quad (2.2)$$

It is straightforward to see that the solutions take now the form

$$1. \quad \hat{\mathbf{g}}, \mathbf{g} \neq \mathbf{0} \quad a(y) = \frac{1}{2} \sqrt{\pm g^2 / \hat{g}^2} \left(e^{\sum_n c_n |y - y_n| + C} \pm e^{-\sum_n c_n |y - y_n| - C} \right), \quad (2.3)$$

where \hat{g} and g are defined as before but now \hat{g} takes the value

$$\hat{g} = \sum_n c_n [2\theta(y - y_n) - 1], \quad (2.4)$$

and g is proportional to \hat{g} with an arbitrary proportionality constant so \hat{g}/g is a true (purely real or imaginary) constant. The simultaneously purely real or imaginary constants c_n are given by

$$c_n = -\frac{\hat{\chi} T_n}{2(\hat{d}-2)} \tanh^{\mp 1} \left(\sum_m c_m |y - y_m| + C \right) \Big|_{y=y_n}. \quad (2.5)$$

$$2. \quad \hat{\mathbf{g}} \neq \mathbf{0}, \mathbf{g} = \mathbf{0} \quad a(y) = e^{\sum_n c_n |y - y_n|}, \quad (2.6)$$

where \hat{g} and the simultaneously purely real constants c_n are given by

$$\hat{g} = \sum_n c_n [2\theta(y - y_n) - 1], \quad c_n = -\frac{\hat{\chi} T_n}{2(\hat{d}-2)}, \quad (2.7)$$

so

$$a(y) = e^{-\frac{\hat{\chi}}{2(\hat{d}-2)} \sum_n T_n |y - y_n|}. \quad (2.8)$$

$$3. \quad \hat{\mathbf{g}} = \mathbf{0} \quad a = \sum_n c_n |y - y_n| + C, \quad (2.9)$$

with

$$g = \sum_n c_n [2\theta(y - y_n) - 1], \quad c_n = -\frac{\hat{\chi} T_n}{2(\hat{d} - 2)} \frac{1}{\sum_m c_m |y - y_m| + C} \Big|_{y=y_n}. \quad (2.10)$$

In general the equations for the constants c_n only have solution if all of them (and, therefore, the tensions T_n) have the same sign. In particular, a system with two branes only has solution if both branes have the same tension. *The exception is the $g = 0$ (RS) case* in which one can get solutions for arbitrary tensions (Eq. (2.8)).

The problem of finding the different c_n 's does not show up if one considers an infinite periodic array of branes and anti-branes with opposite tensions. We can restrict ourselves to a fundamental region bounded by two branes or anti-branes with an anti-brane (resp. brane) in the middle. The system is mirror symmetric with respect to the middle (anti-) brane and we can make a further \mathbb{Z}_2 identification that leaves us with a piece of spacetime bounded by a brane and an anti-brane in which $\hat{\Lambda}$ and Λ are constant (and in which only one constant c_n matters). In these conditions, taking as fundamental region the interval $y \in [0, \ell/2]$ with an anti-brane placed at $y = 0$ and a brane at $y = \ell/2$ the warp function $a(y)$ takes the same form as if there was only one brane in the whole spacetime:

1. $\hat{\mathbf{g}}, \mathbf{g} \neq \mathbf{0}$

$$(a) \quad \hat{\mathbf{g}}, \mathbf{g} \in \mathbb{R} \quad a = g/\hat{g} \cosh(\hat{g}|y| + C), \quad \hat{g} = -\frac{\hat{\chi} T \coth(C)}{\hat{d} - 2}. \quad (2.11)$$

$$(b) \quad \hat{\mathbf{g}} \in \mathbb{R}, \mathbf{g} \in \mathbb{I} \quad a = ig/\hat{g} \sinh(\hat{g}|y| + C), \quad \hat{g} = -\frac{\hat{\chi} T \tanh(C)}{\hat{d} - 2}. \quad (2.12)$$

$$(c) \quad \hat{\mathbf{g}}, \mathbf{g} \in \mathbb{I} \quad a = g/\hat{g} \cos(i\hat{g}|y| + C), \quad \hat{g} = -i\frac{\hat{\chi} T \coth(C)}{\hat{d} - 2}. \quad (2.13)$$

In this case, ℓ must be an integer fraction of the period of y i.e. $\ell = \frac{2\pi}{i\hat{g}}$.

2. $\hat{\mathbf{g}} \neq \mathbf{0}, \mathbf{g} = \mathbf{0}, \hat{\mathbf{g}} \in \mathbb{R}$

$$a = e^{\hat{g}|y|}, \quad \hat{g} = -\hat{\chi} T/2. \quad (2.14)$$

3. $\hat{g} = 0$

$$a = ig|y| + C, \quad g = i \frac{\hat{\chi} TC}{\hat{d} - 2}. \quad (2.15)$$

Let us now consider the bulk and world-brane supersymmetry of these solutions. We can only have supersymmetry on the brane in the RS case $\hat{g} \neq 0, g = 0$ and in the Minkowski-DS case $\hat{g} = 0, g \neq 0$ and imaginary. In these two cases the amount of supersymmetry preserved depends on the d -dimensional (brane) metric $g_{\mu\nu}$. If it is maximally symmetric, then there will be maximal supersymmetry on the brane.

Generic branes generically break \hat{d} -dimensional bulk supersymmetry.⁸ However, in these cases, supersymmetry is not broken *locally* in the bulk, in between any pair of branes, since there the metric has exactly the same form as in the absence of branes.

One may want to have unbroken supersymmetry globally, and not just in between the branes. First, we need to be able to define the Killing-spinor equation globally. In order to do this, we have to allow for a \hat{g} which is piecewise constant instead of globally constant (the main characteristic of these branes is that the value of \hat{g} is different in both sides). We have implicitly accepted this generalization in this section in order to find the solutions. On the other hand, one can use a dual formulation in which the cosmological constant is replaced by a d -form potential as in Ref. [6], an idea which will be investigated in Sec. (4). Once we accept this generalization, the necessary condition to have global unbroken supersymmetry is to be able to match the solutions of the Killing-spinor equation in both sides of a given brane. Let us take, for simplicity, one brane placed at $y = 0$. Both \hat{g} and g change sign across the brane. In the $y > 0$ side of the brane, the solutions of the Killing-spinor equation are those exhibited in the previous section. In the $y < 0$ side of the brane we find solutions of the same form, where, now, the spinors appearing in the general solution satisfy

the same equations but with the sign of g reversed. We need to set $g = 0$ which means that in the second case all supersymmetry is broken unless we have a trivial solution.

In the first case, it is not enough to have $g = 0$ which brings us to the RS case again. It turns out that we also need to impose the condition

$$i\hat{\gamma}_y\hat{\epsilon} = +\hat{\epsilon}, \quad (2.16)$$

on the Killing-spinor, which reduces supersymmetry to a half. This is the same condition we would impose if we were orbifolding the space between branes.

We would like to stress that our results apply strictly to the cases we are considering: the infinitely thin branes described by the above solutions which make the metric across them discontinuous. Thus, our results do not contradict those of Linde and Kallosh [5] who did not study just pure supergravity but included supersymmetric matter. In that paper the authors tried to find supersymmetric *thick* domain walls for which the metric is smooth using consistent superpotentials but did not find any.

⁸We are not going to include sources in the supersymmetry transformation rules as in Ref. [3]. We think one really needs proper κ -symmetric brane-sources in order to study in a fully consistent way the supersymmetric source problem.

2.1 4- d Action and Newton Constant

The action from which the equations of motion Eqs. (2.1) follow has the form

$$\hat{S} = \frac{1}{2\hat{\chi}} \int d^{\hat{d}}\hat{x} \sqrt{|\hat{g}|} \left[\hat{R} - (\hat{d} - 2)\hat{\Lambda} \right] + \text{branes}, \quad (2.17)$$

and for \hat{d} -dimensional metrics of the warped form Eq. (1.1) it reduces to

$$S = \frac{1}{2\hat{\chi}} \int dy a^{\hat{d}-3} \int d^d x \sqrt{|g|} [R - (d - 2)\Lambda]. \quad (2.18)$$

Comparing, we find that the d -dimensional Newton constant χ is related to the \hat{d} -dimensional one $\hat{\chi}$ by

$$\hat{\chi}/\chi = \int dy a^{\hat{d}-3}. \quad (2.19)$$

Taking $\hat{d} = 5$ for definiteness, we can calculate the proportionality factor in the different cases:

Case 1.a: $a = g/\hat{g} \cosh \hat{g}|y|$

$$\chi = 2 \frac{\hat{g}^3/g^2}{\sinh(\hat{g}\ell) + \hat{g}\ell} \hat{\chi}. \quad (2.20)$$

Case 1.b: $a = ig/\hat{g} \sinh |\hat{g}||y|$

$$\chi = 2 \frac{|\hat{g}|^3/(ig)^2}{\sinh(\hat{g}\ell) - \hat{g}\ell} \hat{\chi}. \quad (2.21)$$

Case 1.c: $a = g/\hat{g} \cos i\hat{g}y$

In this case the integration limits are 0 and $2\pi/i\hat{g}$:

$$\chi = \frac{(i\hat{g})^3}{\pi(i\hat{g})^2} \hat{\chi}. \quad (2.22)$$

Case 3: $a = ig|y| + C$

In this case we have:

$$\chi = \frac{3}{2} \frac{ig}{(ig\ell/2 + C)^3 - C^3} \hat{\chi}. \quad (2.23)$$

3 Graviton Dynamics

Expanding the first of Eqs. (2.1) around a background which satisfies the same equation one finds the equation of motion for the perturbation $\hat{h}_{\hat{\mu}\hat{\nu}}$ and using the transverse traceless (tt) gauge

$$\hat{\nabla}^{\hat{\mu}} \hat{h}_{\hat{\mu}\hat{\nu}} = \hat{h} = 0, \quad (3.1)$$

where $\hat{h} = \hat{g}^{\hat{\mu}\hat{\nu}} \hat{h}_{\hat{\mu}\hat{\nu}}$ we get⁹

$$\begin{aligned} & \hat{\nabla}^2 \hat{h}_{\hat{\mu}\hat{\nu}} + 2\hat{R}_{\hat{\rho}(\hat{\mu}} \hat{h}^{\rho}_{\hat{\nu})} + 2\hat{R}^{\hat{\lambda}}_{(\hat{\mu}\hat{\nu})} \hat{h}_{\hat{\lambda}\hat{\sigma}} - 2\hat{\Lambda} \hat{h}_{\hat{\mu}\hat{\nu}} - \\ & + 2\hat{\chi} \left\{ \hat{g}^{\rho\sigma} \left[2\hat{h}_{\rho(\hat{\mu}} \hat{g}_{\hat{\nu})\sigma} - \frac{1}{\hat{d}-2} \left(\hat{h}_{\rho\sigma} \hat{g}_{\hat{\mu}\hat{\nu}} + \hat{h}_{\hat{\mu}\hat{\nu}} \hat{g}_{\rho\sigma} \right) \right] \right. \\ & \left. - \hat{h}^{\rho\sigma} \left(\hat{g}_{\rho\hat{\mu}} \hat{g}_{\sigma\hat{\nu}} - \frac{1}{\hat{d}-2} \hat{g}_{\rho\sigma} \hat{g}_{\hat{\mu}\hat{\nu}} \right) \right\} \sum_n T_n \delta(y - y_n) = 0. \end{aligned} \quad (3.2)$$

Using now the first of Eqs. (2.1) to eliminate $\hat{R}_{\hat{\rho}\hat{\mu}}$ we get

$$\begin{aligned} & \hat{\nabla}^2 \hat{h}_{\hat{\mu}\hat{\nu}} + 2\hat{R}^{\hat{\lambda}}_{(\hat{\mu}\hat{\nu})} \hat{h}_{\hat{\lambda}\hat{\sigma}} + 2\hat{\chi} \left[\left(\hat{g}^{\rho\sigma} \hat{h}_{\rho(\hat{\mu}} - \hat{h}^{\rho\sigma} \hat{g}_{\rho(\hat{\mu}} \right) \hat{g}_{\hat{\nu})\sigma} \right. \\ & \left. + \frac{1}{\hat{d}-2} \left(\hat{h}^{\rho\sigma} \hat{g}_{\rho\sigma} - \hat{g}^{\rho\sigma} \hat{h}_{\rho\sigma} \right) \hat{g}_{\hat{\mu}\hat{\nu}} \right] \sum_n T_n \delta(y - y_n) = 0. \end{aligned} \quad (3.3)$$

Further, using the RS gauge

$$\hat{h}_{\mu y} = \hat{h}_{yy} = 0, \quad (3.4)$$

and the fact that the warped general metric Eq. (1.1) is block-diagonal we see that the source terms vanish identically. The equations for $\hat{h}_{\mu y}, \hat{h}_{yy}$ are satisfied identically and do not become constraints. The equation for the remaining piece of the perturbation is

$$\begin{aligned} & a^{-2} \left[\nabla^2 \hat{h}_{\mu\nu} + 2R^{\rho}_{(\mu\nu)} \hat{h}_{\rho\sigma} \right] - \hat{h}''_{\mu\nu} \\ & - (\hat{d} - 5) a^{-1} a' \hat{h}'_{\mu\nu} + 2[(\hat{d} - 4) a^{-2} (a')^2 + a^{-1} a''] \hat{h}_{\mu\nu} = 0. \end{aligned} \quad (3.5)$$

Now we assume that the perturbation can be expanded in RS modes

$$\hat{h}_{\mu\nu}(x, y) = \sum_{\alpha} f_{\alpha}(y) h_{\mu\nu}^{(\alpha)}(x), \quad (3.6)$$

of which we only keep the massless one $h_{\mu\nu}^{(0)} \equiv h_{\mu\nu}$. The sourceless equation of a massless graviton in a maximally symmetric background, in the tt gauge is

$$\nabla^2 h_{\mu\nu} + 2R^{\rho}_{(\mu\nu)} h_{\rho\sigma} = 0, \quad (3.7)$$

and, thus, we get for $\hat{h}_{\mu\nu} = f_0(y) h_{\mu\nu}$

$$\hat{h}''_{\mu\nu} = (5 - \hat{d}) a^{-1} a' \hat{h}'_{\mu\nu} + 2 \left[a^{-1} a'' + (\hat{d} - 4) a^{-2} (a')^2 \right] \hat{h}_{\mu\nu}. \quad (3.8)$$

⁹All indices are raised and lowered with the full \hat{d} -dimensional background metric $\hat{g}_{\hat{\mu}\hat{\nu}}$.

which in $\hat{d} = 5$ is solved by

$$f_0(y) = a^2(y) . \quad (3.9)$$

Depending on the specific solution we can have gravity confinement on the brane or not. In general, the inclusion of branes and the orbifolding procedure is necessary to have confinement on just one brane (a^2 has more than one maximum in the interval of interest). The only exception seems to be the RS case. The DS to DS case (\hat{g}, g imaginary) deserves special mention because the holographic coordinate is naturally compact. No branes are needed to make the graviton wave-function normalizable, although we do need them if we want to think in terms of confinement. Some of the general results for a metric of the form (1.1) have also been obtained in Ref. [7].

4 A Brane action for the Randall-Sundrum Scenario

A constant can be understood as the dual of a volume-form field strength. A volume-form (*i.e.* a \hat{d} -form which we will denote by $\hat{F}_{(\hat{d})}$) is the field strength of a $d \equiv (\hat{d} - 1)$ -form potential $\hat{A}_{(d)}$. The equation of motion forces the dual of $\hat{F}_{(\hat{d})}$ to be constant (or, more generally, piecewise constant). Thus, one can generically substitute a constant in an action, and, in particular the cosmological constant, by a d -form potential $\hat{A}_{(d)}$. The canonical example is the rewriting of Romans' massive 10-dimensional type IIA supergravity, which contains a mass parameter m , by a 9-form Ramond-Ramond potential to which the D8-brane couples [6].

This implies a generalization of the theory since now one can have solutions in which the value of the cosmological constant is different in different regions of the spacetime: this is precisely what RS-like solutions need. The discontinuities are d -dimensional topological defects (domain walls) which act as sources for the d -form potential and can be interpreted as the worldvolumes of $(\hat{d} - 2)$ -branes charged under the d -form potential (D8-branes in the case of Ref. [6]). A worldvolume action for these branes should, therefore, contain a Wess-Zumino term: the integral of the pullback of the d -form potential.

All this seems to work very well in the D8-brane case and, in fact, the rewriting in terms of a 9-form potential proves necessary and even crucial. It is natural to try something similar here. Therefore, we propose an action consisting in a bulk action containing gravity, $\hat{g}_{\hat{\mu}\hat{\nu}}$, coupled to a d -form potential $\hat{A}_{(d)}$ and a bunch of standard worldvolume actions of $(\hat{d} - 2)$ -branes containing the above-mentioned WZ terms with dynamical coordinate fields $\hat{X}_n^{\hat{\mu}}$, *i.e.*

$$\begin{aligned}
\hat{S} &= \frac{1}{2\chi} \int d^{\hat{d}} \hat{x} \sqrt{|\hat{g}|} \left[\hat{R} + \frac{(-1)^{\hat{d}-2}}{2 \cdot \hat{d}!} \hat{F}_{(\hat{d})}^2 \right] \\
&+ \sum_n \left\{ -\frac{T_n}{2} \int d^d \xi_n \sqrt{|\gamma_n|} \left[\gamma_n^{ij} \partial_i \hat{X}_n^{\hat{\mu}} \partial_j \hat{X}_n^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}(\hat{X}_n) - (\hat{d} - 3) \right] \right. \\
&\left. + \frac{(-1)^d \mu_n}{d!} \int d^d \xi_n \hat{A}_{(d) \hat{\mu}_1 \dots \hat{\mu}_d}(\hat{X}_n) \partial_{i_1} \hat{X}_n^{\hat{\mu}_1} \dots \partial_{i_d} \hat{X}_n^{\hat{\mu}_d} \epsilon^{i_1 \dots i_d} \right\}.
\end{aligned} \tag{4.1}$$

The field configurations that minimize this action satisfy the equations of motion for the metric

$$\begin{aligned}
\hat{G}^{\hat{\mu}\hat{\nu}} + \frac{(-1)^{(\hat{d}-2)} \mu_n}{2 \cdot \hat{d}!} \left[\hat{F}_{(\hat{d})}^{\hat{\mu}\hat{\rho}_1 \dots \hat{\rho}_d} \hat{F}_{(\hat{d})}^{\hat{\nu} \hat{\rho}_1 \dots \hat{\rho}_d} - \frac{1}{2\hat{d}} \hat{g}^{\hat{\mu}\hat{\nu}} \hat{F}_{(\hat{d})}^2 \right] + \\
+ \frac{\chi}{\sqrt{|\hat{g}|}} \sum_n T_n \int d^d \xi_n \sqrt{|\gamma_n|} \gamma_n^{ij} \partial_i \hat{X}_n^{\hat{\mu}} \partial_j \hat{X}_n^{\hat{\nu}} \delta^{\hat{d}}(\hat{x} - \hat{X}_n) = 0,
\end{aligned} \tag{4.2}$$

the d -form potential

$$\hat{\nabla}_{\hat{\mu}} \hat{F}_{(\hat{d})}^{\hat{\mu}\hat{\rho}_1 \dots \hat{\rho}_d} + \frac{2\chi}{\sqrt{|\hat{g}|}} \sum_n \mu_n \int d^d \xi_n \epsilon^{i_1 \dots i_d} \partial_{i_1} \hat{X}_n^{\hat{\mu}_1} \dots \partial_{i_d} \hat{X}_n^{\hat{\mu}_d} \delta^{\hat{d}}(\hat{x} - \hat{X}_n) = 0, \tag{4.3}$$

the worldvolume metric (after some manipulations)

$$\gamma_{nij} - \partial_i \hat{X}_n^{\hat{\mu}} \partial_j \hat{X}_n^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}(\hat{X}_n) = 0, \tag{4.4}$$

and the (coordinate) worldvolume scalars

$$\begin{aligned}
\nabla^2(\gamma) \hat{X}_n^{\hat{\mu}} + \hat{\Gamma}_{\hat{\rho}\hat{\sigma}}^{\hat{\mu}}(\hat{g}) \partial_i \hat{X}_n^{\hat{\rho}} \partial_j \hat{X}_n^{\hat{\sigma}} \gamma_n^{ij} + \\
+ \frac{(-1)^d \mu_n}{T_n d! \sqrt{|\gamma_n|}} \hat{F}_{(\hat{d})}^{\hat{\mu} \hat{\rho}_1 \dots \hat{\rho}_d} \partial_{i_1} \hat{X}_n^{\hat{\rho}_1} \dots \partial_{i_d} \hat{X}_n^{\hat{\rho}_d} \epsilon^{i_1 \dots i_d} = 0.
\end{aligned} \tag{4.5}$$

Eq. (4.4) simply states that the worldvolume metrics are those induced on the worldvolumes by the embedding coordinates $\hat{X}_n^{\hat{\mu}}$. Using worldvolume reparametrization invariance we can set d coordinates (static gauge) to the values $\hat{X}_n^{\hat{\mu}} = \delta_i^{\hat{\mu}} \xi_n^i$. Furthermore, our Ansatz for the remaining coordinate is

$$\hat{X}_n^{\hat{d}} \equiv Y_n = y_n, \tag{4.6}$$

where the y_n 's are constants. We can perform the volume integrals in Eqs. (4.2,4.3) leaving only 1-dimensional delta functions $\delta(y - y_n)$. Also, this implies for the worldvolume metrics (identifying worldvolume and d -dimensional spacetime indices)

$$\gamma_{n\mu\nu} = \hat{g}_{\mu\nu} = a^2(y_n)g_{\mu\nu}. \quad (4.7)$$

For the potential we have

$$\hat{A}_{(d)\mu_1\cdots\mu_d} = ca^d \frac{\epsilon_{\mu_1\cdots\mu_d}}{\sqrt{|g|}}, \Rightarrow \hat{F}_{(\hat{d})\underline{y}\mu_1\cdots\mu_d} = cda^d \log' a \frac{\epsilon_{\mu_1\cdots\mu_d}}{\sqrt{|g|}}, \quad (4.8)$$

where c is a constant to be found and ϵ is the d -dimensional Levi-Civita tensor calculated with the d -dimensional metric.

Let us solve Eqs. (4.5). The equations for the \hat{X}_n^μ 's are automatically solved. $\hat{F}_{(\hat{d})}$ only contributes to the equations of the Y_n 's, which are solved for

$$c = -T_n/\mu_n \equiv T/\mu. \quad (4.9)$$

This implies that all the quotients T_n/μ_n must have the same value, which is a characteristic of BPS objects. Observe that the μ_n 's cannot vanish: had we tried uncharged brane sources we would have never succeeded.

The equation for the potential becomes

$$T/\mu d \log'' a + 2\chi \sum_n \mu_n \delta(y - y_n) = 0, \quad (4.10)$$

which is solved by a warp factor of the RS type

$$a = e^{-\frac{\chi\mu/T}{d} \sum_n \mu_n |y - y_n|}. \quad (4.11)$$

To solve the Einstein equations we first calculate the energy-momentum tensor of the form potential, *i.e.*

$$\hat{F}_{(\hat{d})}^{\hat{\mu}\hat{\rho}_1\cdots\hat{\rho}_d} \hat{F}_{(\hat{d})}^{\hat{\nu}\hat{\rho}_1\cdots\hat{\rho}_d} - \frac{1}{2\hat{d}} \hat{g}^{\hat{\mu}\hat{\nu}} \hat{F}_{(\hat{d})}^2 = \left(\frac{\chi}{2} \sum_n \mu_n [\theta(y - y_n) - 1] \right)^2 \hat{g}^{\hat{\mu}\hat{\nu}}, \quad (4.12)$$

which describes a piecewise cosmological constant. For only two branes with opposite tensions the Einstein equation is exactly the one in Ref. [2] in the intervals in which the cosmological constant is constant and therefore admits the same solutions. In fact, assuming that the d -dimensional metric is Ricci-flat $R_{\mu\nu} = 0$, the Einstein equations are solved by the above warp factor if $(\mu/T)^2 = \frac{1}{2}d/(\hat{d} - 2)$ which implies, as in Sec. (2) and Refs. [2], that

$$a = e^{-\frac{\chi}{(\hat{d}-2)} \sum_n T_n |y - y_n|}. \quad (4.13)$$

5 Conclusions

We have explored general solutions with warped metrics with and without branes and we have studied their supersymmetry properties and the effective theories on the branes, including supersymmetry.

Two cases are singled out by supersymmetry considerations: the well-known RS case and the case in which the total spacetime is Minkowski and on the brane one has DS spacetime. The brane breaks a half of the available supersymmetry in the RS case, a result also obtained in Ref. [3], while in the last case a brane seems to break completely the bulk supersymmetry although one can still speak of (DS) supersymmetry on the brane-world with its known problems.

In analogy with the D8-brane effective action, we considered a formulation of the problem in terms of a dynamical $\hat{d} - 2$ brane with a $\hat{d} - 1$ form potential.

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